

Econ 802

First Midterm Exam

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All questions have equal weight. If something is unclear, please ask. You may want to work first on the questions where you feel most confident.

1. Consider the production possibilities set
$$Y = \{(y_1, y_2, y_3): y_1 \leq 0, y_2 \geq 0, y_3 \leq 0 \text{ and } y_2 \leq -ay_1 - by_3\}$$
 where $a > 0$ and $b > 0$.
 - (a) In the short run $y_3 = k < 0$ is fixed. Draw a graph of the restricted production possibilities set for (y_1, y_2) . Is this set convex? Strictly convex? Explain briefly.
 - (b) Consider price vectors of the form $p = (p_1, p_2, p_3) > 0$. Describe the price vectors for which the firm's short run profit maximization problem has a solution, and the price vectors for which no solution exists. Justify your answer.

2. At the prices $p^A = (1, 3)$ a firm chooses the production plan $y^A = (-3, 1)$. At prices $p^B = (3, 1)$ the same firm chooses the production plan $y^B = (1, -4)$.
 - (a) Show that these data are consistent with the Weak Axiom of Profit Maximization.
 - (b) Draw a graph of some production possibilities set Y for this firm that is consistent with the data. Assume Y is convex and monotonic, and that the quantity of either or both goods could be negative. Can Y include the origin? Why or why not?

3. Consider the profit function $\pi(p, w) = Ap^{1/(1-\alpha)}w^{-\alpha/(1-\alpha)}$ where $A > 0$ and $0 < \alpha < 1$.
 - (a) Show that this function has the usual properties of a profit function (you don't need to prove it is a convex function of prices, but verify the other properties).
 - (b) Find the firm's output supply function $y(p, w)$ and input demand function $x(p, w)$. Then show that the comparative static effects of p and w have the usual signs.

4. A firm has the production function $y = x_1^2 + x_2^2$.
- (a) Say whether this function has constant, increasing, or decreasing returns to scale, and explain why.
 - (b) Derive the conditional input demand functions $x_1(w_1, w_2, y)$ and $x_2(w_1, w_2, y)$. Hint: don't use calculus. Instead, construct a typical isoquant and think about the slope of the isocost lines. Justify your answer carefully.
5. A firm has the production function $y = x_1 x_2$.
- (a) Using standard Lagrangian methods, find the conditional input demand functions $x_1(w, y)$ and $x_2(w, y)$. You can ignore the constraint $x \geq 0$ (don't use Kuhn-Tucker multipliers).
 - (b) Prove that this firm's profit maximization problem has no solution.